Electric Fields and Electric Potential

Unit: Electric Force, Field & Potential

NGSS Standards/MA Curriculum Frameworks (2016): HS-PS3-1, HS-PS3-2, HS-PS3-5

AP® Physics 2 Learning Objectives/Essential Knowledge (2024): 10.3.A, 10.3.A.1, 10.3.A.2, 10.3.A.2.i, 10.3.A.2.ii, 10.3.A.2.iii, 10.3.A.3, 10.3.A.3.i, 10.3.A.3.ii, 10.3.A.3.iii, 10.3.B, 10.3.B.1, 10.3.B.1.i, 10.3.B.1.ii, 10.3.B.2, 10.4.A, 10.4.A.1, 10.4.A.2, 10.4.A.3, 10.5.A, 10.5.A.1, 10.5.A.2, 10.5.A.3, 10.5.A.3.i, 10.5.A.4

Mastery Objective(s): (Students will be able to…)

- Sketch electric field lines and vectors around charged particles or objects.
- Solve problems involving the forces on a charge due to an electric field.

Success Criteria:

- Sketches show arrows pointing from positive charges to negative charges.
- Variables are correctly identified and substituted correctly into the correct part of the correct equation.
- Algebra is correct and rounding to appropriate number of significant figures is reasonable.

Language Objectives:

- Explain how the electric force on a charged particle changes as you get closer to or farther away from another charged object.

Tier 2 Vocabulary: charge, field

Labs, Activities & Demonstrations:

- students holding copper pipe in one hand and zinc-coated steel pipe in other—measure with voltmeter. (Can chain students together.)

Notes:

force field: a region in which an object experiences a force because of some intrinsic property of the object that enables the force to act on it. Force fields are vectors, which means they have both a magnitude and a direction.

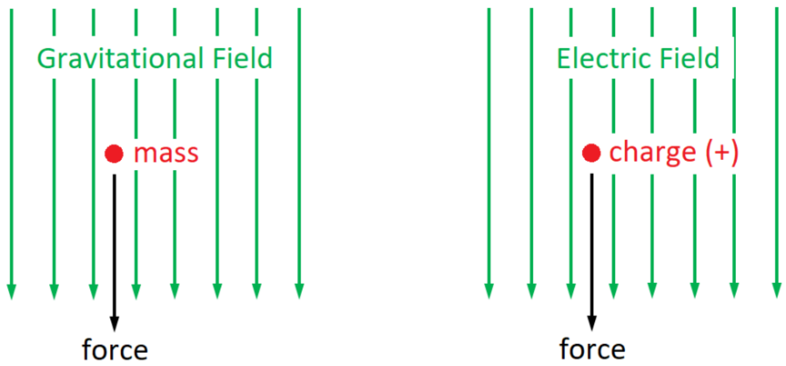
electric field ($\vec{E}$) : an electrically charged region (force field) that exerts a force on any charged particle within the region.

An electric field applies a force to an object based on its electrical charge. $\vec{F}_e = q\vec{E}$, where $\vec{E}$ represents the magnitude and direction of the electric field.

Because gravity is a familiar concept, it is useful to use gravitational fields as a way to explain force fields, and thus electric fields.

Recall that a gravitational field applies a force to an object based on its mass. $\vec{F}_g = m\vec{g}$, where $\vec{g}$ represents the magnitude and direction of the gravitational field.

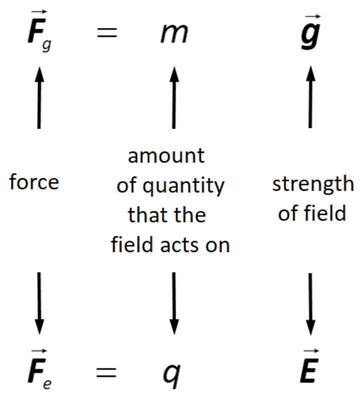
Just as a gravitational field applies a force to an object that has mass, an electric field applies a force to an object that has charge:



A key difference between the two situations is that there are two kinds of charges—positive and negative—whereas there is only one kind of mass.

The force on an object with mass is always in the direction of the gravitational field. However, the direction of the force on an object with charge depends on whether the charge is positive or negative. *The force on an object with **positive charge** is in the **same direction** as the electric field; the force on an object with **negative charge** is always in the **opposite direction** from the electric field.*

For any force field, the amount of force is the amount of the quantity that the field acts on times the strength of the field:

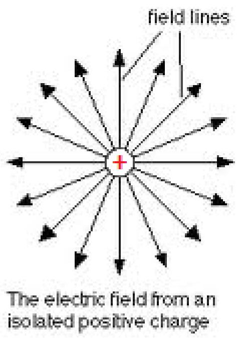


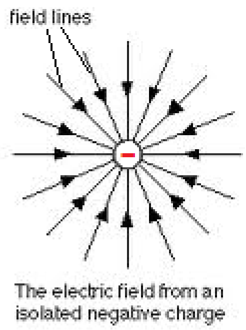
<u>field lines</u>: lines with arrows that show the direction of an electric field. In the above diagrams, the arrows are the field lines.

Field lines are lines that show the directions of force on an object. As described above, for an electric field, the object is assumed to be a positively charged particle. This means that *the direction of the electric field is from positive to negative.* This means that field lines go outward in all directions from a positively charged particle, and inward from all directions toward a negatively-charged particle.

This means that a positively-charged particle (such as a proton) would move in the direction of the arrows, and a negatively charged particle (such as an electron) would move in the opposite direction.

The simplest electric field is the region around a single charged particle:



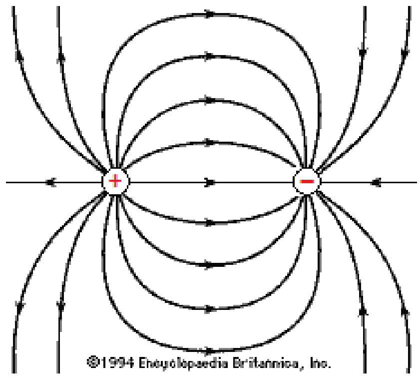


As stated in the *Electric Charge* topic (starting on page 153), the charges in a solid conductor repel one another, resulting in the charges moving to the outside of the conductor. This means that *the electric field inside of a conductor is zero.*

However, the same is not true for insulators. If you have an insulator (such as a dielectric) in an electric field, the excess electric charge is spread throughout the insulator, and the electric field can have a nonzero value.

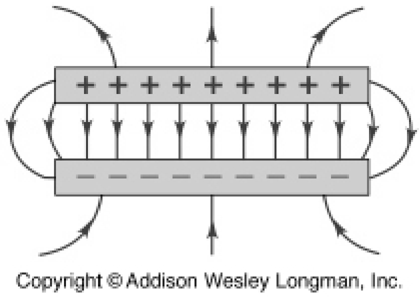
If you have a hollow conducting sphere (such as a hollow metal ball), the sphere will conduct the charges to the outside, and the electric field inside of the sphere will be zero. In this situation, the sphere may be considered as a point charge, as if all of the charge were placed at its center.

If a positive and a negative charge are near each other, the field lines go from the positive charge toward the negative charge:

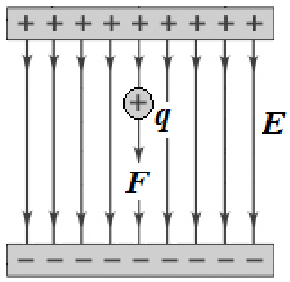


(Note that even though this is a two-dimensional drawing, the field itself is three-dimensional. Some field lines come out of the paper from the positive charge and go into the paper toward the negative charge, and some go behind the paper from the positive charge and come back into the paper from behind toward the negative charge.)

In the case of two charged plates (flat surfaces), the field lines would look like the following:



Electric Field Strength



We can measure the strength of an electric field by placing a particle with a positive charge (q) in the field and measuring the force ($\vec{F}$) on the particle.

Coulomb's Law tells us that the force on the charge is due to the charges from the electric field:

$$F_e = \frac{kq_1q_2}{r^2}$$

If the plates have equal charge densities, the repulsive force from the like-charged plate decreases as the particle moves away from it, but the attractive force from the oppositely charged plate increases by the same amount as the particle moves toward it.

This means that if the positive and negative charges on the two surfaces that make the electric field have equal charge densities, *the force is the same everywhere in between the two surfaces*. The force on the particle is related only to the strength of the electric field and the charge of the particle.

This results in the equation that defines the electric field ($\vec{E}$) as the force between the electric field and our particle, divided by the charge of our particle:

$$\vec{E} = \frac{\vec{F}}{q} \quad \text{or} \quad \vec{F} = q\vec{E}$$

Electric Potential Energy and Electric Potential

Recall from the work-energy theorem that work equals a change in energy. Because an electric field can do work on a charged particle, an electric field must therefore apply energy to the particle.

electric potential energy (U_e): the amount of energy required to either bring two opposite charges together, or drive two like charges apart to a distance of infinity (*i.e.,* a distance at which they would no longer exert a force on each other).

Note that potential energy of any kind (including electric potential energy) is a property of a system. Objects within a system can only have potential energy with respect to each other.

Recall from physics 1 that energy added or released is work (by definition). This means that the potential energy of an object with respect to another object in a system is the force required to bring the objects together times the distance over which that force would need to act:

$$U_e = \vec{\boldsymbol{F}}_e \bullet \vec{\boldsymbol{d}}$$

Because the distance can be in any direction, it makes sense to think of the distance as a radius *r,* using spherical coordinates. This means, according to Coulomb's Law:

$$U_e = F_e \cdot r = \frac{kq_1q_2}{r^2} \cdot r = \frac{kq_1q_2}{r}$$

electric potential (V): the electric potential of a location in space is the amount of work that would be required to bring a unit positive charge (*i.e.,* an object with a charge of +1 C) from "infinity" (which really means from some location that has an electric potential of zero) to that location. *I.e.,* if q = 1 C, then $V = U_e$.

For a charge of any other amount, q, its electric potential would be U_e divided by q:

$$V = \frac{U_e}{q} = \frac{kq}{r}$$

Therefore, the electric potential at a point due to multiple charges would be:

$$V = k\sum_i \frac{q_i}{r_i} \quad \text{or, because } k = \frac{1}{4\pi\varepsilon_o}, \quad V = \frac{1}{4\pi\varepsilon_o} \cdot \sum_i \frac{q_i}{r_i}$$

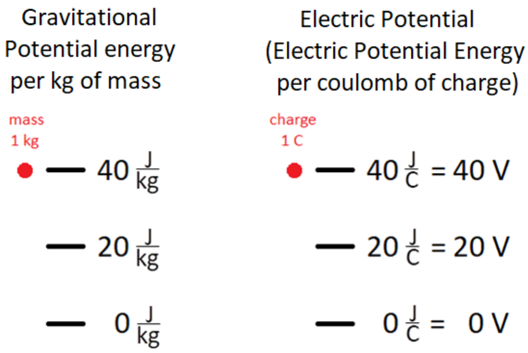
Electric potential is measured in volts (V).

$$1\,\text{V} \equiv 1\,\tfrac{\text{J}}{\text{C}} \equiv 1\,\tfrac{\text{N·m}}{\text{C}}$$

Electric potential energy is analogous to gravitational potential energy. In a gravitational field, a particle has gravitational potential energy because gravity can make it move. In an electric field, a particle has electric potential (energy) because the electric field can make it move.

For example, if we had a 1 kg mass and we placed it at a height of 4 m above the ground, its gravitational potential would be $U_g = mgh = (1)(10)(4) = 40\text{ J}$.

Similarly, if we put an object with a charge of 1 C at a location that has an electric potential of 40 V, that object would have 40 J of potential energy due to the electric field.



$$\frac{U_g}{m} = \vec{\boldsymbol{g}} \bullet \vec{\boldsymbol{h}}$$

gravitational potential energy per unit of mass

$$V = \frac{U_e}{q} = \frac{W}{q} = \vec{\boldsymbol{E}} \bullet \vec{\boldsymbol{d}}$$

electric potential
(already per unit of charge)

Work Done on a Charge by an Electric Field

Recall from physics 1 that work is the dot product of force and displacement:

$$W = \vec{F} \cdot \vec{d} = Fd\cos\theta$$

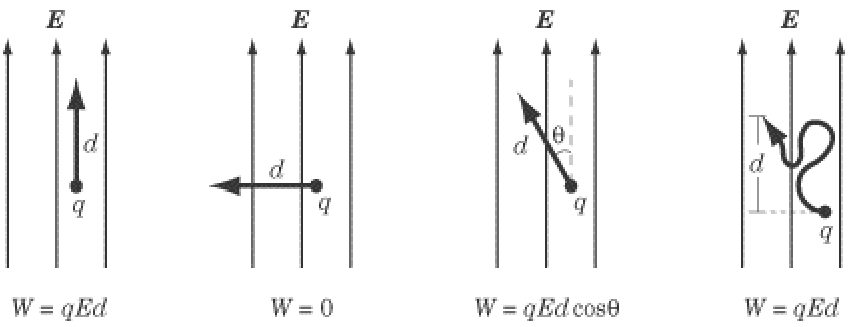
Because $W = \Delta U$, the potential energy of a charge in an electric field is the work that the field is able to do on the charge. This means:

$$U_e = \vec{F}_e \cdot \vec{d} = \frac{kq_1q_2}{r^2} \cdot r = \frac{kq_1q_2}{r}$$

Because $\vec{F} = q\vec{E}$, we can substitute:

$$W = \vec{F} \bullet \vec{d} = q\,\vec{E} \bullet \vec{d} = qEd\cos\theta$$

Note that the electric field can only cause displacement in the direction of the field. Therefore, the work caused by the electric field depends only on the displacement in the direction of the field. (This is where the $\cos\theta$ term comes from.)



A more familiar term in electricity is voltage.

voltage = potential difference (ΔV): the difference in electric potential between two points in space. This difference is caused by the action of an electric field at a distance:

$$\Delta V = \frac{\Delta U_e}{q} = \frac{W}{q} = \vec{\boldsymbol{E}} \bullet \vec{\boldsymbol{d}} = Ed\cos\theta$$

Note that many physics texts use the variable *V* for both electric potential and voltage (potential difference), and the reader needs to figure out which one is meant from context. To eliminate (or at least reduce) the confusion that this causes, these notes will use *V* for electric potential and Δ*V* for potential difference (voltage).

Because voltage is simply a difference between two electric potentials, potential difference (voltage) is measured in volts (V).

Because the work done by an electric field is $W = qEd\cos\theta$ and potential difference is $\Delta V = Ed\cos\theta$, we can substitute ΔV for $Ed\cos\theta$, which gives:

$$W = q\Delta V$$

Redistribution of Charges

When conductors are placed in electrical contact, electrons will redistribute themselves so that the surfaces of the conductors have the same electric potential. This makes sense—because electrons repel each other, if they are able to move (*i.e.,* in a conductor), they will spread out to minimize the repulsion.

This is why birds can sit on power lines, even if those power lines are made of bare copper wire. Because the wire conducts the charges freely, the electrons spread out to equalize the distance, which means the potential difference between any one point on the wire and another is the same. This means there is no potential difference between the part of the wire where one of a bird's legs contacts it and part of the wire where the other touches. Because the wire conducts electricity much better than the bird does, virtually no electric charge goes through the bird.

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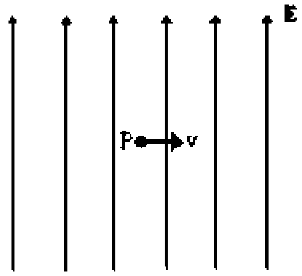
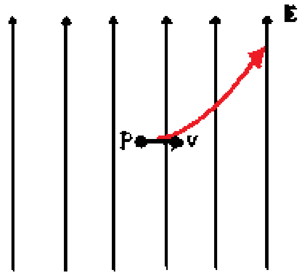
Sample Problem:



Q: A proton has a velocity of $1\times10^{5}\ \frac{\text{m}}{\text{s}}$ when it is at point *P* in a uniform electric field that has an intensity of $1\times10^{4}\ \frac{\text{N}}{\text{C}}$. Calculate the force (magnitude and direction) on the proton and sketch its path.



A: The force on the proton is given by:

$$\vec{F}_e = q\vec{E} = (1.6\times10^{-19})(1\times10^{4}) = 1.6\times10^{-15}\text{ N}$$

The direction of the force is the same direction as the electric field, which in this problem is upwards.

An upward force causes acceleration upwards. Because the proton starts with a velocity only to the right, upward acceleration means that its velocity will have a continuously increasing vertical component.

Homework Problems

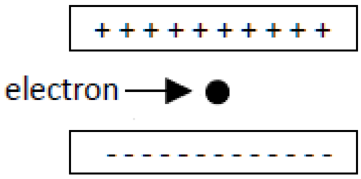
1. **(M)** Sketch the electric field in all directions around each of the following charged particles. (Assume that each particle has the same amount of charge.)

   a. 

   b. 

   c. 

2. **(M)** An electron is placed exactly halfway between two charged parallel plates, as shown in the diagram at the right. The electric field strength between the plates is $4.8\times10^{-11}\,\frac{\text{N}}{\text{C}}$.



   a. Sketch field lines to represent the electric field between the plates.

   b. Which direction does the electron move?

   c. As the electron moves, does the force acting on it increase, decrease, or remain the same?

   d. What is the net force on the electron?

   Answer: -7.68×10^{-30} N
   (Negative means the opposite direction of the electric field.)