Electric Current & Ohm's Law

Unit: DC Circuits

NGSS Standards/MA Curriculum Frameworks (2016): HS-PS2-9(MA)

AP® Physics 2 Learning Objectives/Essential Knowledge (2024): 11.3.A, 11.3.A.1, 11.3.A.2, 11.3.A.2.i, 11.3.A.2.ii, 11.3.B, 11.3.B.1, 11.3.B.1.i, 11.3.B.1.ii, 11.3.B.1.iii, 11.3.B.1.iv, 11.4.A, 11.4.A.1

Mastery Objective(s): (Students will be able to…)

- Solve problems involving relationships between voltage, current, resistance and power.

Success Criteria:

- Variables are correctly identified and substituted correctly into the correct equation.
- Algebra is correct and rounding to appropriate number of significant figures is reasonable.

Language Objectives:

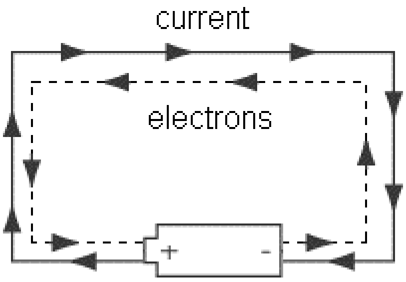
- Describe the relationships between voltage, current, resistance, and power.

Tier 2 Vocabulary: current, resistance, power

Labs, Activities & Demonstrations:

- modeling resistivity with straws
- make a light bulb out of a pencil "lead" (graphite)

Notes:



electric current (I): the flow of charged particles through a conductor, caused by a difference in electric potential. The direction of the electric current is defined as the direction that a positively-charged particle would move. Note, however, that the particles that are actually moving are electrons, which are negatively charged.

This means that electric current "travels" in the *opposite* direction from the electrons. We will use conventional current (pretending that positive particles are flowing through the circuit) throughout this course.

Electric current ($\vec{I}$) is a vector quantity and is measured in amperes (A), often abbreviated as "amps". One ampere is one coulomb per second.

$$I = \frac{\Delta Q}{t}$$

Note that when electric current is flowing, charged particles move from where they are along the circuit. For example, when a light bulb is illuminated, the electrons that do the work for the first few minutes are already in the filament.

voltage (potential difference) (ΔV)*: the difference in electric potential energy between two locations, per unit of charge.

$$\Delta V = \frac{W}{q}$$

Potential difference is the work (W) done on a charge per unit of charge (q). Potential difference (ΔV) is a scalar quantity (in DC circuits) and is measured in volts (V), which are equal to joules per coulomb.

The total voltage in a circuit is usually determined by the power supply that is used for the circuit (usually a battery in DC circuits).

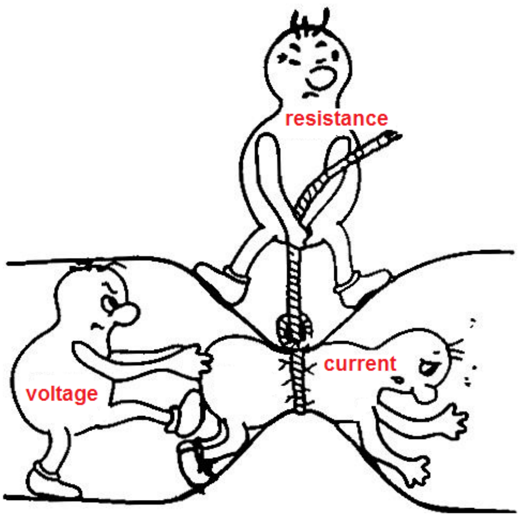
resistance (R): the amount of electromotive force (electric potential) needed to force a given amount of current through an object in a DC circuit.

$$R = \frac{\Delta V}{I}$$

Resistance (R) is a scalar quantity and is measured in ohms (Ω). One ohm is one volt per ampere.

This relationship is Ohm's Law, named for the German physicist Georg Ohm. Ohm's Law is more commonly written:

$$I = \frac{\Delta V}{R} \quad \text{or} \quad \Delta V = IR$$



Simply put, Ohm's Law states that an object has an ability to resist electric current flowing through it. The more resistance an object has, the more voltage you need to force electric current through it. Or, for a given voltage, the more resistance an object has, the less current will flow through it.

Resistance is an intrinsic property of a substance. In this course, we will limit problems that involve calculations to ohmic resistors, which means their resistance does not change with temperature.

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* Note that most physics texts (and most physicists and electricians) use V for both electric potential and voltage, and students have to rely on context to tell the difference. In these notes, to make the distinction clear (and to be consistent with the AP® Physics 2 exam), we will use V for electric potential, and ΔV for voltage (potential difference).

Choosing the voltage and the arrangement of objects in the circuit (which determines the resistance) is what determines how much current will flow.

Electrical engineers use resistors in circuits to reduce the amount of current that flows through the components.

Every physical object has resistance.

- Substances that are good conductors have minimal resistance. *The resistance of wires is small enough that it can be ignored, unless the wire is the only element of the circuit.*
- Substances that are good insulators have very large resistance. Air, for example, has a resistance of 10^{12} to $10^{16}\ \frac{\Omega}{\text{cm}}$; it takes about 21 100 V to create a spark that can bridge a gap of 1 cm of air. This means that *an air gap is considered to be an open circuit, in which no current flows*.

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<u>impedance</u> (Z): the opposition that a circuit presents to a current when a voltage is applied. In a DC circuit, impedance and resistance are equivalent. In an AC circuit, the oscillating voltage creates changing electric and magnetic fields, which themselves resist the changes caused by the alternating current. This means the opposition to current is constantly changing at the same frequency as the oscillation of the current.

Mathematically, impedance is represented as a complex number, in which the real part is resistance and the imaginary part is reactance, a quantity that takes into account the effects of the oscillating electric and magnetic fields.

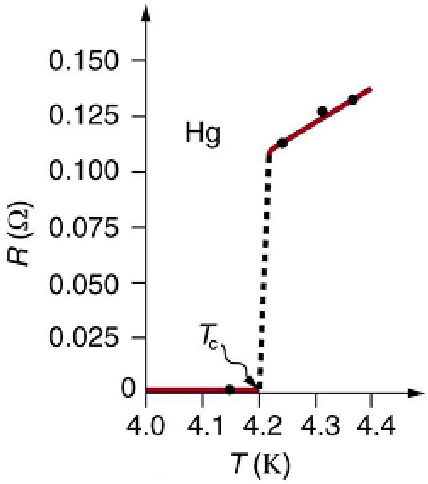
<u>resistivity (ρ)</u>: the innate ability of a substance to offer electrical resistance. The resistance of an object is therefore a function of the resistivity of the substance (ρ), and of the length (L) and cross-sectional area (A) of the object. In MKS units, resistivity is measured in ohm-meters (Ω·m).

$$R = \frac{\rho L}{A}$$

Resistivity changes with temperature. For small temperature differences (less than 100°C), resistivity is given by:

$$\rho = \rho_o(1 + \alpha \Delta T)$$

where ρ_o is the resistivity at some reference temperature and α is the temperature coëfficient of resistivity for that substance. For conductors, α is positive (which means their resistivity increases with temperature). For metals at room temperature, resistivity typically varies from +0.003 to +0.006 K^{-1}.



Some materials become superconductors (essentially zero resistance) at very low temperatures. The temperature below which a material becomes a superconductor is called the critical temperature (T_c). For example, the critical temperature for mercury is 4.2 K, as shown in the graph to the right.

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conductivity (σ): the innate ability of a substance to conduct electricity. Conductivity (σ) is the inverse of resistivity, and is measured in siemens (S). Siemens used to be called mhos (symbol ℧). (Note that "mho" is "ohm" spelled backwards.)

$$\sigma = \frac{1}{\rho}$$

ohmic resistor: a resistor whose resistance is the same regardless of voltage and current. The filament of an incandescent light bulb is an example of a non-ohmic resistor, because the current heats up the filament, which increases its resistance. (This is necessary in order for the filament to also produce light.)

capacitance (C): the ability of an object to hold an electric charge. Capacitance (C) is a scalar quantity and is measured in farads (F). One farad equals one coulomb per volt.

$$C = \frac{Q}{\Delta V}$$

power (P): as discussed in the mechanics section of this course, power (P) is the work done per unit of time and is measured in watts (W).

In electric circuits:

$$P = \frac{W}{t} = I\Delta V = I^2 R = \frac{(\Delta V)^2}{R}$$

work (W): recall from mechanics that work (W) equals power times time, and is measured in either newton-meters (N·m) or joules (J):

$$W = Pt = I\Delta V t = I^2 R t = \frac{(\Delta V)^2 t}{R} = Vq$$

Electrical work or energy is often measured in kilowatt-hours (kW·h).

$$1\,\text{kW}\cdot\text{h} \equiv 3.6\times10^{6}\ \text{J} \equiv 3.6\,\text{MJ}$$

Summary of Terms, Units and Variables

Term	Variable	Unit	Term	Variable	Unit
point charge	q	coulomb (C)	resistance	R	ohm (Ω)
charge	Q	coulomb (C)	capacitance	C	farad (F)
current	I	ampere (A)	power	P	watt (W)
voltage	ΔV	volt (V)	work	W	joule (J)

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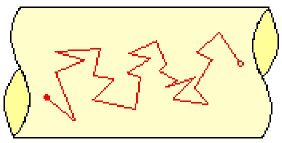
Alternating Current *vs.* Direct Current

Electric current can move in two ways.

direct current: electric current flows through the circuit, starting at the positive terminal of the battery or power supply, and ending at the negative terminal. Batteries supply direct current. A typical AAA, AA, C, or D battery supplies 1.5 volts DC.

However, the net flow of charged particles through a wire is very slow. Electrons continually collide with one another in all directions as they drift slowly through the circuit. Individual electrons in a DC circuit have a net velocity of about one meter per hour.



alternating current: electric current flows back and forth in one direction and then the other, like a wave. The current alternates at a particular frequency. In the U.S., household current is 110 – 120 volts AC with a frequency of 60 Hz. In most of the rest of the world, household current is 230 volts AC with a frequency of 50 Hz.

Alternating current requires higher voltages in order to operate devices, but has the advantage that the voltage drop is much less over a length of wire than with direct current.

Sample Problems:

Q: A simple electrical device uses 1.5 A of current when plugged into a 110 V household electrical outlet. How much current would the same device draw if it were plugged into a 12 V outlet in a car?

A: Resistance is a property of a specific object. Because we are not told otherwise, we assume the device is ohmic and the resistance is the same regardless of the current.

Therefore, our strategy is to use the information about the device plugged into a household outlet to determine the device's resistance, then use the resistance to determine how much current it draws in the car.

In the household outlet:

$$R = \frac{\Delta V}{I} = \frac{110}{1.5} = 73.\overline{3}\,\Omega$$

In the car:

$$I = \frac{\Delta V}{R} = \frac{12}{73.\overline{3}} = 0.163\text{ A}$$

Q: A laptop computer uses 10 W of power. The laptop's power supply adjusts the current so that the power is the same regardless of the voltage supplied. How much current would the computer draw from a 110 V household outlet? How much current would the same laptop computer need to draw from a 12 V car outlet?

A: The strategy for this problem is the same as the previous one.

Household outlet:

$$P = I\Delta V$$

$$I = \frac{P}{\Delta V} = \frac{10}{110} = 0.091\text{ A}$$

Car outlet:

$$I = \frac{P}{\Delta V} = \frac{10}{12} = 0.8\overline{3}\text{ A}$$

Q: A 100 Ω resistor is 0.70 mm in diameter and 6.0 mm long. If you wanted to make a 470 Ω resistor out of the same material (with the same diameter), what would the length need to be? If, instead, you wanted to make a resistor the same length, what would the new diameter need to be?

A: In both cases, $R=\frac{\rho L}{A}$.

For a resistor of the same diameter (same cross-sectional area), ρ and A are the same, which means:

$$\frac{R'}{R}=\frac{L'}{L}$$

$$L'=\frac{R'L}{R}=\frac{(470)(6.0)}{100}=28.2\,\text{mm}$$

For a resistor of the same length, ρ and L are the same, which means:

$$\frac{R'}{R}=\frac{A}{A'}=\frac{\pi r^2}{\pi (r')^2}=\frac{\pi(d/2)^2}{\pi (d'/2)^2}=\frac{d^2}{(d')^2}$$

$$d'=\sqrt{\frac{Rd^2}{R'}}=d\sqrt{\frac{R}{R'}}=0.70\sqrt{\frac{100}{470}}=0.70\sqrt{0.213}=0.323\,\text{mm}$$

Homework Problems

1. **(S)** An MP3 player uses a standard 1.5 V battery. How much resistance is in the circuit if it uses a current of 0.010 A?

   Answer: 150 Ω

2. **(M)** How much current flows through a hair dryer plugged into a 110 V circuit if it has a resistance of 25 Ω?

   Answer: 4.4 A

3. **(S)** A battery pushes 1.2 A of charge through the headlights in a car, which has a resistance of 10 Ω. What is the potential difference across the headlights?

   Answer: 12 V

4. **(M)** A circuit used for electroplating copper applies a current of 3.0 A for 16 hours. How much charge is transferred?

   Answer: 172 800 C

5. **(S)** What is the power when a voltage of 120 V drives a 2.0 A current through a device?

   Answer: 240W

6. **(S)** What is the resistance of a 40. W light bulb connected to a 120 V circuit?

   Answer: 360 Ω

7. **(M)** If a component in an electric circuit dissipates 6.0 W of power when it draws a current of 3.0 A, what is the resistance of the component?

   Answer: 0.67 Ω

8. **(S)** A 0.7 mm diameter by 60 mm long pencil "lead" is made of graphite, which has a resistivity of approximately $1.0\times10^{-4}\ \Omega\cdot\text{m}$. What is its resistance?

   Hints:

   - *You will need to convert mm to m.*
   - *You will need to convert the diameter to a radius before using $A=\pi r^2$ to find the area.*

   Answer: 15.6 Ω

9. **(M)** A cylindrical object has radius *r* and length *L* and is made from a substance with resistivity *ρ*. A potential difference of Δ*V* is applied to the object. Derive an expression for the current that flows through it.

   Hint: this is a two-step problem.

   Answer: $I=\dfrac{(\Delta V)A}{\rho L}$

10. **(S)** Some children are afraid of the dark and ask their parents to leave the hall light on all night. Suppose the hall light in a child's house has two 75. W incandescent light bulbs (150 W total), the voltage is 120 V, and the light is left on for 8.0 hours.

    a. How much current flows through the light fixture?

    Answer: 1.25 A

    b. How many kilowatt-hours of energy would be used in one night?

    Answer: 1.2 kW·h

    c. If the power company charges 22 ¢ per kilowatt-hour, how much does it cost to leave the light on overnight?

    Answer: 26.4 ¢

    d. If the two incandescent bulbs are replaced by LED bulbs that use 12.2 W each (24.4 W total) how much would it cost to leave the light on overnight?

    Answer: 4.3 ¢