Ideal Gas Law

Unit: Thermodynamics

MA Curriculum Frameworks (2016): HS-PS2-8(MA)

AP® Physics 2 Learning Objectives: 3.4.C.1, 3.4.C.2, 4.C.3.1, 7.A.3.3, 7.B.1.1

Mastery Objective(s): (Students will be able to…)

- Describe the relationship between any two variables in the ideal gas law.
- Apply the ideal gas law to problems involving a sample of gas.

Success Criteria:

- Solutions have the correct quantities substituted for the correct variables.
- Chosen value of the gas constant has the same units as the other quantities in the problem.
- Algebra and rounding to appropriate number of significant figures is correct.

Language Objectives:

- Identify each quantity based on its units and assign the correct variable to it.
- Explain the placement of each quantity in the ideal gas law.

Tier 2 Vocabulary: ideal, law

Notes:

ideal gas: a gas that behaves according to Kinetic-Molecular Theory (KMT).

When we developed the combined gas law, before we cancelled the number of moles or particles, we had the equations:

$$\frac{P_1V_1}{n_1T_1} = \frac{P_2V_2}{n_2T_2} = \frac{PV}{nT} = R \text{ (constant)} \qquad \frac{P_1V_1}{N_1T_1} = \frac{P_2V_2}{N_2T_2} = \frac{PV}{NT} = k_B \text{ (constant)}$$

using moles — using particles

where n is the number of moles of gas, and N is the number of gas particles. One mole is 6.02×10^{23} particles, which means $N = (6.02 \times 10^{23})n$

Because P, V, n and T are all of the quantities needed to specify the conditions of an ideal gas, this expression must be true for *any ideal gas* under *any conditions*. If V is in m^3, P is in Pa, n is in moles, and T is in Kelvin, then:

$$R = 8.31 \tfrac{\text{J}}{\text{mol·K}} \quad \text{and} \quad k_B = 1.38 \times 10^{-23} \tfrac{\text{J}}{\text{K}}$$

R is called "the gas constant," and k_B is Boltzmann's constant.

We can rearrange $\frac{PV}{nT} = R$ and $\frac{PV}{NT} = k_B$ to get the ideal gas law in its familiar form:

$$PV = nRT \quad \text{and} \quad PV = Nk_BT$$

Use this space for summary and/or additional notes:

Other Values of R

The purpose of the gas constant *R* is to convert the quantity *nT* from units of mol·K to units of pressure × volume. This constant can have different values, depending on the units that it needs to cancel:

$$\mathrm{R} = 8.31\,\frac{\mathrm{J}}{\mathrm{mol{\cdot}K}} \equiv 8.31\,\frac{\mathrm{m^3{\cdot}Pa}}{\mathrm{mol{\cdot}K}} \equiv 8.31\,\frac{\mathrm{L{\cdot}kPa}}{\mathrm{mol{\cdot}K}} \equiv 8.31\times10^{-3}\,\frac{\mathrm{kJ}}{\mathrm{mol{\cdot}K}}$$

$$\mathrm{R} = 0.0821\,\frac{\mathrm{L{\cdot}atm}}{\mathrm{mol{\cdot}K}} \qquad \mathrm{R} = 62.4\,\frac{\mathrm{L{\cdot}torr}}{\mathrm{mol{\cdot}K}} \qquad \mathrm{R} = 1.987\,\frac{\mathrm{cal}}{\mathrm{mol{\cdot}K}} \equiv 1.987\,\frac{\mathrm{BTU}}{\mathrm{lb\text{-}mol{\cdot}^{\circ}R}}$$

Use of non-S.I. units, such as atm or torr, is more common in chemistry. In this course, we will use the S.I. units of m^3 for volume and Pa for pressure. The unit $Pa{\cdot}m^3$ is equivalent to a joule.

Solving Problems Using the Ideal Gas Law

If a gas behaves according to the ideal gas law, simply substitute the values for pressure, volume, number of moles (or particles), and temperature into the equation. Be sure your units are correct (especially that temperature is in Kelvin), and that you use the correct constant, depending on whether you know the number of particles or the number of moles of the gas.

Sample Problem:

A 3.50 *n* mol sample of an ideal gas has a pressure of *P* 120 000 Pa and a temperature of 35 °C. What is its volume?

$T \rightarrow K$ (*V*)

Answer:

Note that because pressure is given in pascals (Pa), we need to use the value of the gas constant that also uses Pa: $R = 8.31\,\frac{\mathrm{m^3 \cdot Pa}}{\mathrm{mol \cdot K}}$

P = 120 000 Pa

V = V

n = 3.50 mol

$R = 8.31\,\frac{\mathrm{m^3 \cdot Pa}}{\mathrm{mol \cdot K}}$

T = 35 °C + 273 = 308 K

Then we substitute these numbers into the ideal gas law and solve:

$$PV = nRT$$

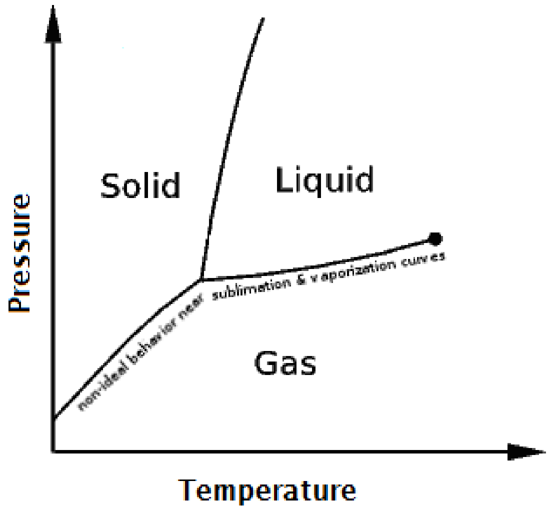
$$V = \frac{nRT}{P} = \frac{(3.50)(8.31)(308)}{120\,000} = 0.0747\,\mathrm{m^3}$$

Use this space for summary and/or additional notes:

honors (not AP®)

Real Gases

As stated previously, when the temperature and pressure of a gas are close to the solid or liquid regions of the phase diagram for the substance, gases start to exhibit non-ideal behaviors. Recall the following definition of a real gas:



real gas: a gas whose molecules do ***not*** behave according to kinetic-molecular theory (KMT). This occurs most commonly at temperatures and pressures that are close to the solid or liquid regions of the phase diagram for the substance.

In the late 19th century, the Dutch physicist Johannes van der Waals published a correction to the ideal gas law that can be applied to real gases.

The van der Waals Equation applies correction factors to the pressure and volume terms in the equation:

$$\left(P+a\frac{n^2}{V^2}\right)(V-nb)=nRT$$

in this equation, the constants *a* and *b* are properties specific to a gas and must be looked up or determined experimentally.

The corrected pressure term ($P+a\frac{n^2}{V^2}$ instead of P) is because molecules attract each other slightly at low pressures, but repel each other when they are forced close together. This repulsion acts like additional pressure.

The corrected volume term ($V-nb$ instead of V) term is because the ideal gas law assumes that the molecules are far enough apart that we do not need to consider the volumes of the molecules themselves as part of the volume of their container. As the molecules are brought closer together, we have to subtract the space taken up by *n* moles of molecules from the available volume.

You will not need to solve problems using the van der Waals equation in this course.

Use this space for summary and/or additional notes:

Homework Problems

Use the ideal gas law to solve the following problems. Be sure to choose the appropriate value for the gas constant and to convert temperatures to Kelvin.

1. **(M)** A sample of 1.00 moles of oxygen at 50.0 °C and 98.6 kPa occupies what volume?

   Answer: 27.2 L

2. **(S)** If a steel cylinder with a volume of 1.50 L contains 10.0 moles of oxygen, under what pressure is the oxygen if the temperature is 27.0 °C?

   Answer: 164 atm = 125 000 torr = 16 600 kPa

3. **(S)** In a gas thermometer, the pressure of 0.500 L of helium is 113.30 kPa at a temperature of −137 °C. How many moles of gas are in the sample?

   Answer: 0.050 mol

4. **(M)** A sample of 4.25 mol of hydrogen at 20.0 °C occupies a volume of 25.0 L. Under what pressure is this sample?

   Answer: 4.09 atm = 3 108 torr = 414 kPa

Use this space for summary and/or additional notes: