DC Resistor-Capacitor (RC) Circuits

Unit: DC Circuits

NGSS Standards/MA Curriculum Frameworks (2016): N/A

AP® Physics 2 Learning Objectives/Essential Knowledge (2024): 11.8.B, 11.8.B.1, 11.8.B.1.i, 11.8.B.1.ii, 11.8.B.1.iii, 11.8.B.2, 11.8.B.2.i, 11.8.B.2.ii, 11.8.B.2.iii, 11.8.B.2.iv, 11.8.B.2.v, 11.8.B.2.vi, 11.8.B.2.vii

Mastery Objective(s): (Students will be able to…)

- Solve problems involving time-varying circuits with charging and discharging capacitors.

Success Criteria:

- Correct relationships are applied for each quantity.
- Variables are correctly identified and substituted correctly into the correct equations and algebra is correct.

Language Objectives:

- Explain why a discharged capacitor behaves like a wire, and why a fully-charged capacitor behaves like an open switch.

Tier 2 Vocabulary: charge, capacitance, resistance

Labs, Activities & Demonstrations:

- RC circuit lab

Notes:

RC circuit: a circuit involving combinations of resistors and capacitors.

In an RC circuit, the amount and direction of current change with time as the capacitor charges or discharges. The amount of time it takes for the capacitor to charge or discharge is determined by the combination of the capacitance and resistance in the circuit. This makes RC circuits useful for intermittent (*i.e.,* with a built-in delay) back-and-forth switching. Some common uses of RC circuits include:

- clocks
- windshield wipers
- pacemakers
- synthesizers

When we studied resistor-only circuits, the circuits were steady-state, *i.e.,* voltage and current remained constant. RC circuits are time-variant, *i.e.,* the voltage, current, and charge stored in the capacitor(s) are all changing with time.

Charging a Capacitor

Recall that a capacitor is an electrical component that stores charge. No current actually flows through the capacitor. Recall also that capacitance (C) is a capacitor's ability to be charged by a given electric potential difference (voltage). Therefore, the maximum charge that a capacitor can hold is:

$$Q_{max} = C\Delta V$$

In the previous section, the charge that we calculated was actually this maximum charge Q_{max}, which is the amount of charge that the capacitor would hold if it had been charged for "a long time" such that it was fully charged.

However, recall also that:

- In a capacitor with zero charge, every charge placed on one side causes an equivalent charge on the opposite side. This means:

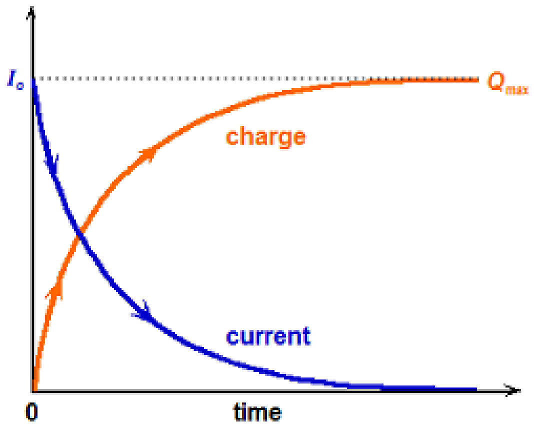
  With respect to current, a capacitor with zero charge initially behaves like a wire.

- When a capacitor is fully charged, no additional charge can be added (unless the voltage is increased). This means:

  With respect to current, a fully charged capacitor behaves like an open circuit.

This means that the behavior of the capacitor changes as the charges build up inside of it.

When a capacitor that initially has zero charge is connected to a voltage source, the current that flows through the circuit decreases exponentially, and the charge stored in the capacitor asymptotically approaches Q_{max}, the maximum charge that can be stored in that capacitor for the voltage applied.



(Note that the graphs are not to scale; the y-axis scale and units are necessarily different for charge and current.)

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The equations for the charge in a capacitor and the current that flows "through" it as a function of time while a capacitor is charging are:

$$I = I_o e^{-t/RC} = \frac{\Delta V}{R} e^{-t/RC}$$
$$Q = Q_{max}(1 - e^{-t/RC}) = C\Delta V(1 - e^{-t/RC})$$

where:

- I = current (A)
- I_o = initial current (just after switch was closed) (A)
- ΔV = voltage (V)
- Q = charge (C)
- Q_{max} = (theoretical) maximum charge stored by capacitor at the circuit's voltage (C)
- e = base of exponential function = 2.71828…
- t = time since switch was closed (s)
- R = resistance (Ω)
- C = capacitance (F)

We can rearrange the above equations to give:

$$\frac{I}{I_o} = 1 - \frac{Q}{Q_{max}} = e^{-t/RC}$$

Note that the value *RC* is the time constant of the circuit, often denoted by the variable τ. Thus, we can write the above equation as:

$$\frac{I}{I_o} = 1 - \frac{Q}{Q_{max}} = e^{-t/\tau}$$

The RC term in the exponent is known as the time constant (τ) for the circuit. Larger values of RC mean the circuit takes longer to charge the capacitor. The following table shows the rate of decrease in current in the charging circuit and the rate of increase in charge on the capacitor as a function of time:

t	$\frac{I}{I_o} = \frac{\Delta V}{\Delta V_o} = e^{-t/RC}$	$\frac{Q}{Q_{\max}} = 1 - e^{-t/RC}$
0	1	0
¼ RC	0.78	0.22
½ RC	0.61	0.39
0.69 RC	0.5	0.5
RC	0.37	0.63
2 RC	0.14	0.86
4 RC	0.02	0.98
10 RC	4.5×10^{-5}	≈ 1

Note that the half-life of the charging (and discharging) process is approximately 0.69 RC.

Note also that while Q_{max} depends on the voltage applied, the rate of charging and discharging depend only on the resistance and capacitance in the circuit.

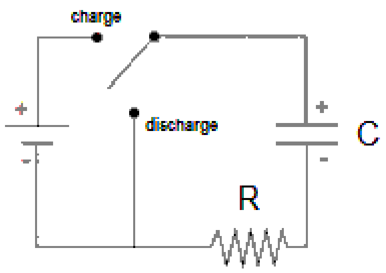
The AP® Physics 2 exam requires only a qualitative understanding of RC circuits. It is sufficient to understand that:

- The time constant $\tau = RC$ represents how quickly a capacitor will charge or discharge.
- After a "long time" (approximately 10τ), a capacitor that is charging can be assumed to be fully charged, and a capacitor that is discharging can be assumed to be fully discharged.
- After an amount of time equal to the time constant $(t = \tau = RC)$, a fully discharged capacitor will charge to approximately 63 % of its full capacity.
- After an amount of time equal to the time constant $(t = \tau = RC)$, a fully charged capacitor will discharge to approximately 37 % of its initial charge.

Discharging a Capacitor

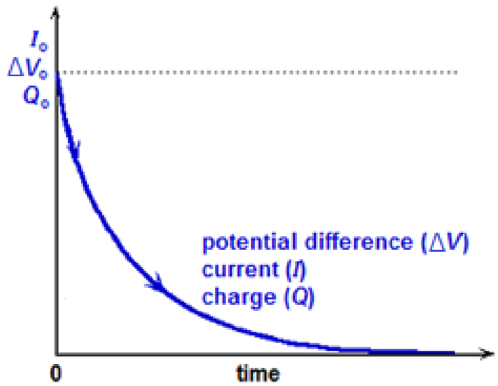
Just as a capacitor charges gradually, it also discharges gradually.

Imagine we have a circuit like the one at the right.



When the switch is in the "charge" position, the battery charges the capacitor. When the switch is flipped to the "discharge" position, the battery is switched out and the circuit contains only the capacitor and the resistor.

When this happens, the capacitor discharges (loses its charge). The capacitor acts as a temporary voltage source, and current temporarily flows out of the capacitor through the resistor.



(Note again that the graphs are not to scale; the y-axis scale and units are necessarily different for current, voltage and charge.)

The driving force for this temporary current is the repulsion from the stored charges in the capacitor. As charge leaves the capacitor there is less repulsion, which causes the voltage and current to decrease exponentially along with the charge.

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The equations for discharging a capacitor are therefore identical in form to the equations for charging it:

$$\Delta V = \Delta V_o e^{-t/RC} \qquad Q = Q_o e^{-t/RC} \qquad I = I_o e^{-t/RC}$$

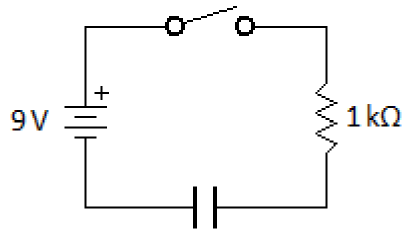
or, dividing each by its value at time zero:

$$\frac{\Delta V}{\Delta V_o} = \frac{Q}{Q_o} = \frac{I}{I_o} = e^{-t/RC}$$

Again the time constant, RC, is the relative amount of time it takes for the charge remaining in the capacitor and the voltage and current in the circuit to decay. (Refer to the table on page 269.)

Sample Problem

Q: A circuit has a 9 V battery, an open switch, a 1 kΩ resistor, and a capacitor in series. The capacitor has no residual charge.



When the switch is closed, the charge in the capacitor climbs to 86 % of its maximum value in 50 ms. What is the capacitance of the capacitor?

A: The charge increases at the rate of:

$$Q(t) = Q_{max}(1 - e^{-t/RC})$$

We are given that $\frac{Q}{Q_{max}} = 0.86$, $t = 0.05\,\text{s}$, and $R = 1000\,\Omega$.

$$\frac{Q}{Q_{max}} = 1 - e^{-t/RC}$$

$$0.86 = 1 - e^{-0.05/1000\,C}$$

$$-0.14 = -e^{-0.05/1000\,C}$$

$$\ln(0.14) = \ln(e^{-0.05/1000\,C}) = \frac{-0.05}{1000\,C}$$

$$-1.97 = \frac{-0.05}{1000\,C}$$

$$1970\,C = 0.05$$

$$C = \frac{0.05}{1970} = 2.5 \times 10^{-5}\,\text{F} = 25\,\mu\text{F}$$

Note that we could have solved this problem by using the table on page 269 to see that the capacitor is 86 % charged $\left(\frac{Q}{Q_{max}} = 0.86\right)$ when $t = 2\,RC$.

Homework Problems

Note: You will need to convert units in most of these problems.

1. **(S)** A series RC circuit consists of a 9 V battery, a 3 Ω resistor, a 6 μF capacitor and a switch. How long would it take after the switch is closed for the capacitor to reach 63 % of its maximum potential difference?

   Answer: 18 μs

2. **(M)** A circuit contains a 9 V battery, an open switch, a 1 kΩ resistor, and a capacitor, all in series. The capacitor initially has no charge. When the switch is closed, the charge on the capacitor climbs to 86 % of its maximum value in 50 ms. What is the capacitance of the capacitor?

   Answer: 25 μF

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3. **(S)** A heart defibrillator has a capacitance of 25 µF and is charged to a potential difference of 350 V. The electrodes of the defibrillator are attached to the chest of a patient who has suffered a heart attack. The initial current that flows out of the capacitor is 10 mA.

    a. How much time does it take for the current to fall to 0.5 mA?

    Answer: 2.62 s

    b. How much charge is left on the defibrillator plates after 1.2 s?

    Answer: 2.22 mC

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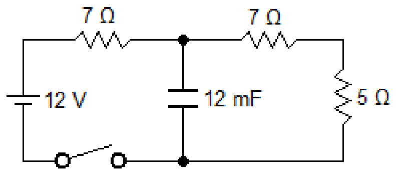
4. **(M)** In the following RC circuit:



the switch (S) has been closed for a long time.

a. When the switch is opened, how much time does it take for the charge on the capacitor to drop to 13.5% of its original value?

Answer: 0.288 s

b. What is the maximum current through the 5 Ω resistor the instant the switch is opened?

Answer: 0.63 A