Magnetism & Moving Charges

Unit: Magnetism & Electromagnetism

NGSS Standards/MA Curriculum Frameworks (2016): HS-PS2-5

AP® Physics 2 Learning Objectives/Essential Knowledge (2024): 12.2.A, 12.2.A.1, 12.2.A.1.i, 12.2.A.1.ii, 12.2.A.1.iii, 12.2.B, 12.2.B.1, 12.2.B.2, 12.2.B.2.i, 12.2.B.2.ii, 12.2.B.3, 12.2.B.4, 12.3.A, 12.3.A.1, 12.3.A.1.i, 12.3.A.1.ii, 12.3.A.1.iii, 12.3.A.1.iv, 12.3.A.1.v, 12.3.B.1, 12.3.B.1.i, 12.3.B.1.ii

Mastery Objective(s): (Students will be able to…)

- Explain how a moving charge (including current in a wire) creates a magnetic field.
- Explain how a magnetic field exerts a force on a moving charge.
- Solve problems involving moving charges in a magnetic field.
- Solve problems involving the magnetic field produced by a current-carrying wire.
- Solve problems involving the force produced by a current-carrying wire in an external magnetic field.

Success Criteria:

- Equations are set up correctly, with correct variable substitution.

Language Objectives:

- Identify the parts of a diagram or lab/demonstration set-up, and which quantity corresponds to which object.
- Explain how each object in the diagram or set-up contributes to the magnetic field and forces.

Tier 2 Vocabulary: field, wire, current

Labs, Activities & Demonstrations:

- current-carrying wire in a magnetic field
- electromagnet
- two coils wired together and two rare earth magnets, one on a spring
- electric motor
- magnet through copper pipe (Lenz's Law)
- wire & galvanometer jump rope
- neodymium magnet & CRT screen

Notes:

Recall that electrons have both charge and a property called "spin"[*], which is responsible for magnetism, as discussed in *Magnetism & Magnetic Permeability*, starting on page 282.

The interaction of these properties has several consequences:

- When an electron moves, both its charge and its magnetic spin are moving. The movement of this magnetic spin creates a magnetic field.
- An electric field is produced by charged objects. (See *Electric Fields and Electric Potential*, starting on page 171.) When these charges move, the electric field changes, and the movement of the charges produces a magnetic field.

Therefore, ***a changing electric field produces a magnetic field***, and ***a changing magnetic field produces an electric field***.

Note that this and the remaining topics in this unit involve the cross-product of vectors and the right-hand rule. It would be useful to review *Vector Multiplication*, starting on page 28.

Forces and Moving Charges

The changing electric field given by the cross-product of the velocity of the charged particle and the magnetic field:

$$\vec{\boldsymbol{E}} = \vec{\boldsymbol{v}} \times \vec{\boldsymbol{B}} \qquad \text{and} \qquad E = vB\sin\theta$$

Because the force is given by $\vec{\boldsymbol{F}} = q\vec{\boldsymbol{E}}$, the force $\vec{\boldsymbol{F}}$ on a point charge q that is moving through a magnetic field $\vec{\boldsymbol{B}}$ with velocity $\vec{\boldsymbol{v}}$ is therefore:

$$\vec{\boldsymbol{F}}_B = q(\vec{\boldsymbol{v}} \times \vec{\boldsymbol{B}}) \qquad \text{and} \qquad F_B = qvB\sin\theta$$

The cross-product means that the direction of the force must be perpendicular to both the velocity of the charged object and the magnetic field.

---

[*] Remember that electrons are not actually spinning. The property was named based on a simpler understanding of the behavior of electrons than we have now.

Magnetism, Forces, and Current-Carrying Wires

Recall that current is a flow of charges, which means that an electric current moving through a magnetic field creates a force on the wire carrying the current.

Recall from *Electric Current & Ohm's Law* starting on page 197, that $\vec{I} = \frac{\Delta q}{t}$. Recall from physics 1 that $\vec{v} = \frac{\vec{d}}{t} = \frac{\ell}{t}$. In this case, we can replace $\vec{d}$ with ℓ, where ℓ is the length (distance) of the section of the wire that passes through the magnetic field.

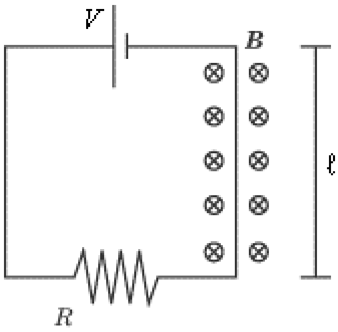
Combining these equations gives $q\vec{v} = \ell\vec{I}$, which we can use to create the equation:

$$\vec{F} = \ell(\vec{I} \times \vec{B}) \qquad \text{and} \qquad F = \ell I B \sin\theta$$

Note that the direction of the cross products $\vec{v} \times \vec{B}$ and $\vec{I} \times \vec{B}$ can be determined using the right-hand rule.

A current passing through a magnetic field would be represented like the diagram at the right.

In this diagram, the battery has voltage *V*, the resistor has resistance *R*, and the length of wire passing through the magnetic field is ℓ. The direction of the current, $\vec{I}$, as it passes through the magnetic field is from the bottom of the page to the top.



The magnetic field has strength ***B***, and is denoted by the symbols ⊗ ⊗ ⊗ ⊗ ⊗ which denote a magnetic field going *into* the page. (A field coming out of the page would be denoted by ⊙ ⊙ ⊙ ⊙ ⊙ instead. Think of the circle as an arrow inside a tube. The dot represents the tip of the arrow facing toward you, and the "X" represents the fletches (feathers) on the tail of the arrow facing away from you.)

By the right-hand rule, the direction of the force on the wire would be to the left.

For example, suppose we were given the following for the above diagram:

$$\vec{B} = 4.0\times10^{-5}\text{ T}$$
$$V = 30\text{ V}$$
$$R = 5\,\Omega$$
$$\ell = 2\text{ m}$$

Current (from Ohm's Law):

$$V = IR$$
$$30 = I(5)$$
$$I = 6\text{A}$$

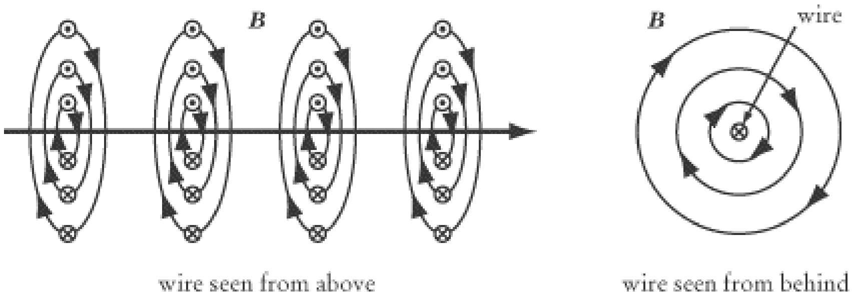
Force on the wire:

$$\vec{F} = \ell(\vec{I}\times\vec{B})$$
$$F = \ell IB\sin\theta$$
$$F = (2)(6)(4.0\times10^{-5})\sin(90^\circ)$$
$$F = (2)(6)(4.0\times10^{-5})(1)$$
$$F = 4.8\times10^{-4}\text{N}$$

If the current is going upward through the magnetic field, and the magnetic field is pointing into the paper, then the right-hand rule tells us that the force would be directed to the left.

Magnetic Field Produced by Electric Current

An electric current moving through a wire also produces a magnetic field around the wire:



This time, we use the right-hand rule with our thumb pointing in the direction of the current, and our fingers curl in the direction of the magnetic field.

The strength of the magnetic field produced is given by the Ampere's Law, named for the French physicist, André-Marie Ampère, who discovered this relationship in the 1820s:

$$B = \frac{\mu_o}{2\pi}\cdot\frac{I}{r}$$

where B is the strength of the magnetic field, μ_o is the magnetic permeability of free space, I is the current, and r is the distance from the wire. (The variable r is used because the distance is in all directions, which means we would use polar or cylindrical coordinates.)

Combined Electric and Magnetic Fields

As stated above, the force on a charged particle due to an electric field is:

$$\vec{F}_e = q\vec{E}$$

and the force on a charged particle due to a magnetic field is:

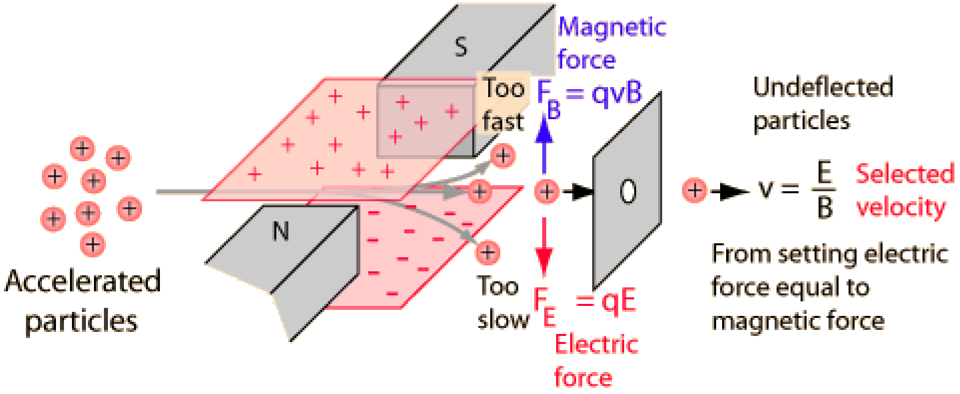
$$\vec{F}_B = q(\vec{v} \times \vec{B})$$

Therefore, the force on a charged particle that interacts simultaneously with an electric field and a magnetic field must be the sum of the two:

$$\vec{F}_{EM} = q(\vec{E} + \vec{v} \times \vec{B})$$

An application of this combination is a particle sorter, which allows only particles with a given velocity to pass through.

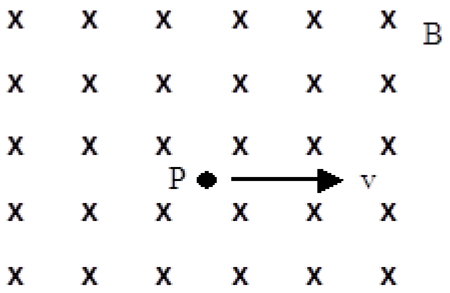
Particles that enter a mass spectrometer must have the correct velocity in order for the mass spectrometer to be able to separate the particles properly. Before the particles enter the mass spectrometer, they first pass through a particle sorter, which applies opposing electric and magnetic forces to the particle:



If the particles are moving too quickly, the magnetic force is stronger and the particles are deflected upwards. If the particles are moving too slowly, the electric force is stronger and the particles are deflected downwards. For particles with the desired velocity, the forces are equal, there is no net force, and the particles are not deflected.

Sample Problem:

Q: A proton has a velocity of $1\times10^{5}\ \frac{\text{m}}{\text{s}}$ to the right when it is at point *P* in a uniform magnetic field of 0.1 T that is directed into the page. Calculate the force (magnitude and direction) on the proton, and sketch its path.



A: The force on the proton is given by:

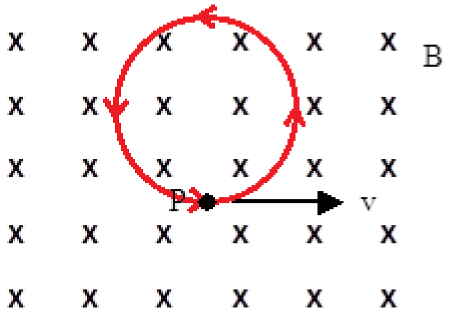
$$\vec{F}_B = q(\vec{v}\times\vec{B}) = qvB\sin\theta$$

Because the velocity of the particle and the direction of the magnetic field are perpendicular, $\sin\theta = \sin(90°) = 1$ and therefore the magnitude of $F_B = qvB$.

$$F_B = qvB$$
$$F_B = (1.6\times10^{-19})(1\times10^{5})(0.1)$$
$$F_B = 1.6\times10^{-15}\,\text{N}$$

The direction is given by the right-hand rule. Start with the fingers of your right hand pointing straight in the direction of velocity (to the right) and rotate your right hand until you can bend your fingers toward the magnetic field (into the page). Your thumb will be pointing upwards, which means that the force on a positively charged particle moving to the right is upwards. (Note that if the particle had been negatively charged, it would have moved in the opposite direction.)

However, note that the action of the force causes a change in the direction of the proton's velocity, and the change in the direction of the proton's velocity changes the direction of the force. This feedback loop results in the proton moving in a continuous circle.



Homework Problems

1. **(M)** A wire 1 m long carries a current of 5 A. The wire is at right angles to a uniform magnetic field. The force on the wire is 0.2 N. What is the strength of the magnetic field?

   Answer: 0.04 T

2. **(S)** A wire 0.75 m long carries a current of 3 A. The wire is at an angle of 30° to a uniform magnetic field. The force on the wire is 0.5 N. What is the strength of the magnetic field?

   Answer: $0.\overline{4}$ T

3. **(M)** Two currents (one of 2 A and the other of 4 A) flow through wires that are parallel to each other, with the currents flowing in the same direction. The wires are 3 m long and are separated by 8 cm (0.08 m). What is the net magnetic field at the midpoint between the two wires?

   (*Hint: Find the magnetic field produced by each wire and add them, remembering that magnetic fields are vectors.*)

   Answer: 1×10^{-5} T

4. **(M)** A wire 2 m long moves perpendicularly through a 0.08 T field at a speed of $7\,\frac{\text{m}}{\text{s}}$.

   a. What emf is induced?

   Answer: 1.12 V

   b. If the wire has a resistance of 0.50 Ω, use Ohm's Law to find the current that flows through the wire.

   Answer: 2.24 A