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Sound & Music

Unit: Mechanical Waves

NGSS Standards/MA Curriculum Frameworks (2016): HS-PS4-1

AP® Physics 2 Learning Objectives/Essential Knowledge (2024): N/A

Mastery Objective(s): (Students will be able to…)

- Describe how musical instruments produce sounds.
- Describe how musical instruments vary pitch.
- Calculate frequencies of pitches produced by a vibrating string or in a pipe.

Success Criteria:

- Descriptions & explanations account for observed behavior.
- Variables are correctly identified and substituted correctly into the correct equations.
- Algebra is correct and rounding to appropriate number of significant figures is reasonable.

Language Objectives:

- Explain what produces the vibrations in various types of musical instruments.

Tier 2 Vocabulary: pitch

Labs, Activities & Demonstrations:

- Show & tell: violin, penny whistle, harmonica, Boomwhackers.
- Helmholtz resonators—bottles of different sizes/air volumes, slapping your cheek with your mouth open.
- Frequency generator & speaker.
- Rubens tube ("sonic flame tube").
- Measure the speed of sound in air using a resonance tube.

Notes:

Sound waves are caused by vibrations that create longitudinal (compressional) waves in the medium they travel through (such as air).

pitch: how "high" or "low" a musical note is. The pitch is determined by the frequency of the sound wave.

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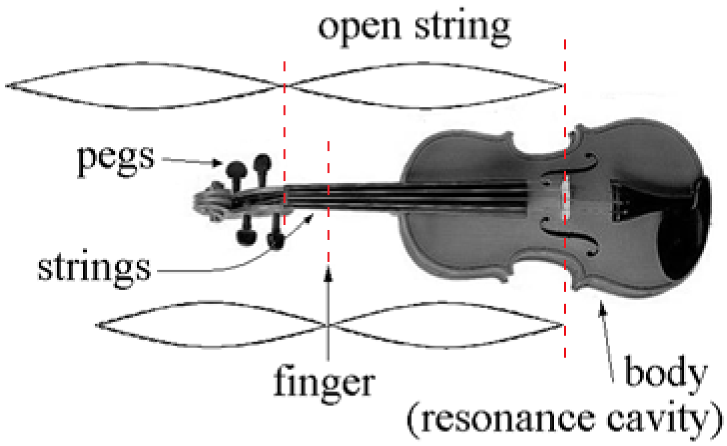
resonance: when the wavelength of a half-wave (or an integer number of half-waves) coincides with one of the dimensions of an object. This creates standing waves that reinforce and amplify each other. The body of a musical instrument is an example of an object that is designed to use resonance to amplify the sounds that the instrument produces.

String Instruments

A string instrument (such as a violin or guitar) typically has four or more strings. The lower strings (strings that sound with lower pitches) are thicker, and higher strings are thinner. Pegs are used to tune the instrument by increasing (tightening) or decreasing (loosening) the tension on each string.



The vibration of the string creates a half-wave, *i.e.,* λ = 2L. The musician changes the half-wavelength by using a finger to shorten the part of the string that vibrates. (A shorter wavelength produces a higher frequency = higher pitch.)

The velocity of the wave produced on a string depends on the tension and the length and mass of the vibrating portion. The velocity is given by the equation:

$$v_{string} = \sqrt{\frac{F_T L}{m}}$$

where:

f = frequency (Hz)

F_T = tension (N)

m = mass of string (kg)

L = length of string (m) $= \frac{\lambda}{2}$

Given the velocity and wavelength, the frequency (pitch) is therefore:

$$f = \frac{v}{\lambda} = \frac{v}{2L} = \frac{1}{2L}\sqrt{\frac{F_T L}{m}} = \sqrt{\frac{F_T}{4mL}}$$

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Pipes and Wind Instruments

A pipe (in the musical instrument sense) is a tube filled with air. The design of the mouthpiece (or air inlet) causes the air to oscillate as it enters the pipe. This causes the air molecules to compress and spread out at regular intervals based on the dimensions of the closed section of the instrument, which determines the wavelength. The wavelength and speed of sound determine the frequency.

Most wind instruments use one of three ways of causing the air to oscillate:

Brass Instruments

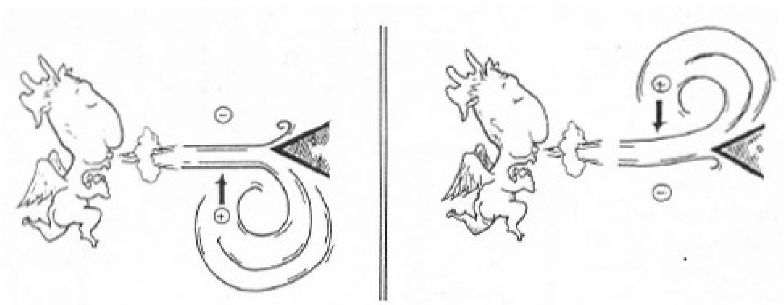
With brass instruments like trumpets, trombones, French horns, *etc.*, the player presses his/her lips tightly against the mouthpiece, and the player's lips vibrate at the appropriate frequency.

Reed Instruments

With reed instruments, air is blown past a reed (a semi-stiff object) that vibrates back and forth. Clarinets and saxophones use a single reed made from a piece of cane (a semi-stiff plant similar to bamboo). Oboes and bassoons ("double-reed instruments") use two pieces of cane that vibrate against each other. Harmonicas and accordions use reeds made from a thin piece of metal.

Whistles (Instruments with Fipples)

Instruments with fipples include recorders, whistles and flutes. A fipple is a sharp edge that air is blown past. The separation of the air going past the fipple results in a pressure difference on one side *vs.* the other. Air moves toward the lower pressure side, causing air to build up and the pressure to increase. When the pressure becomes greater than the other side, the air switches abruptly to the other side of the fipple. Then the pressure builds on the other side until the air switches back:



The frequency of this back-and-forth motion is what determines the pitch.

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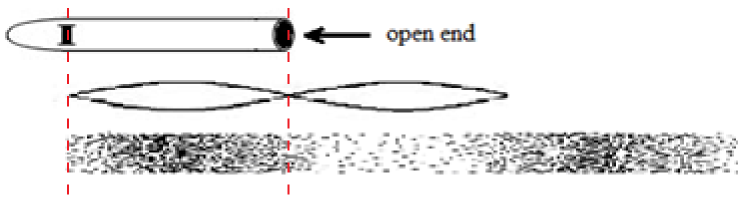
Open vs. Closed-Pipe Instruments

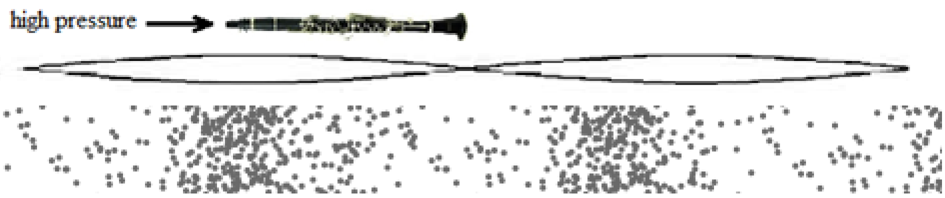
An open-pipe instrument has an opening at each end. A closed-pipe instrument has an opening at one end, and is closed at the other.

Examples of open-pipe instruments include uncapped organ pipes, whistles, recorders and flutes.



Notice that the two openings determine where the air pressure *must* be equal to atmospheric pressure (*i.e.,* the air is neither compressed nor expanded). This means that the length of the body of the instrument (L) is a half-wave, and that the wavelength (λ) of the sound produced must therefore be twice as long, *i.e.,* $\lambda = 2L$. (This is similar to string instruments, in which the length of the vibrating string is a half-wave.)

Examples of closed-pipe instruments include clarinets and all brass instruments. Air is blown in at high pressure via the mouthpiece, which means the mouthpiece is an antinode—a region of maximum displacement of the individual air molecules. This means that the body of the instrument is the distance from the antinode to a region of atmospheric pressure, *i.e.,* one-fourth of a wave. This means that for closed-pipe instruments, $\lambda = 4L$.



The difference in the resonant wavelength ($4L$ *vs.* $2L$) is why a closed-pipe instrument (*e.g.,* a clarinet) sounds an octave lower than an open-pipe instrument of similar length (*e.g.,* a flute)—twice the wavelength results in half the frequency.

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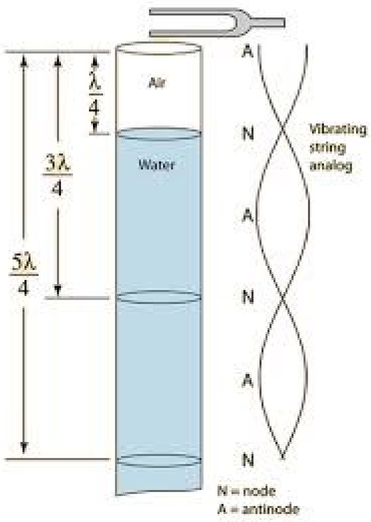
The principle of a closed-pipe instrument can be used in a lab experiment to determine the frequency of a tuning fork (or the speed of sound) using a resonance tube—an open tube filled with water to a specific depth.



A tuning fork generates an oscillation of a precise frequency at the top of the tube. Because this is a closed pipe, the source (just above the tube) is an antinode (maximum amplitude).

When the height of air above the water is exactly ¼ of a wavelength $(\frac{\lambda}{4})$, the waves that are reflected back have maximum constructive interference with the source wave, which causes the sound to be significantly amplified. This phenomenon is called resonance.

Resonance will occur at every antinode—*i.e.,* any integer plus ¼ of a wave ($\frac{\lambda}{4}$, $\frac{3\lambda}{4}$, $\frac{5\lambda}{4}$, *etc.*)

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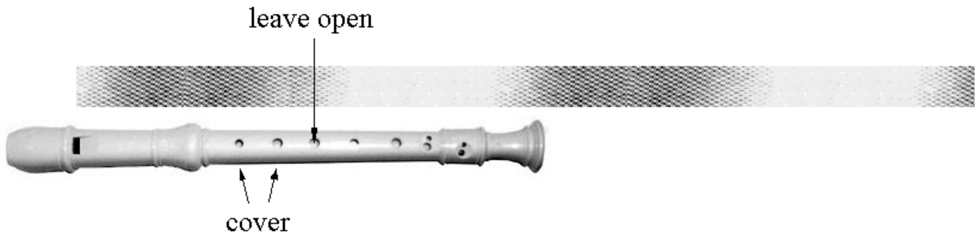
Playing Different Pitches (Frequencies)

For an instrument with holes, like a flute or recorder, the first open hole is the first place in the pipe where the pressure is equal to atmospheric pressure, which determines the half-wavelength (or quarter-wavelength):



The speed of sound in air is v_s ($343\,\frac{m}{s}$ at 20 °C and 1 atm), which means the frequency of the note (from the formula $v_s = \lambda f$) will be:

$f = \frac{v_s}{2L}$ for an open-pipe instrument (*e.g.,* flute, recorder, whistle)

$f = \frac{v_s}{4L}$ for an closed-pipe instrument (*e.g.,* clarinet, brass instrument).

Note that the frequency is directly proportional to the speed of sound in air. The speed of sound increases as the temperature increases, which means that as the air gets colder, the frequency gets lower, and as the air gets warmer, the frequency gets higher. This is why wind instruments go flat at colder temperatures and sharp at warmer temperatures. Musicians claim that the instrument is going out of tune, but actually it's not the instrument that is out of tune, but the speed of sound!

Note however, that the frequency is inversely proportional to the wavelength (which depends largely on the length of the instrument). This means that the extent to which the frequency changes with temperature will be different for different-sized instruments, which means the band will become more and more out of tune with itself as the temperature changes.

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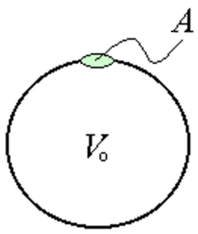
Helmholtz Resonators

The resonant frequency of a bottle or similar container (called a Helmholtz resonator, named after the German physicist Hermann von Helmholtz) is more complicated to calculate.

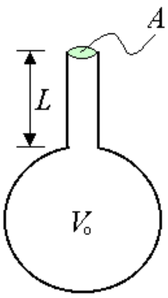
For an enclosed volume of air with a single opening, the resonant frequency depends on the resonant frequency of the air in the large cavity, and the cross-sectional area of the opening.



Resonant frequency:

$$f = \frac{v_s}{2\pi} \cdot \frac{A}{V_o}$$

For a bottle with a neck, the air in the neck behaves like a spring, with a spring constant that is proportional to the volume of air in the neck:



Resonant frequency:

$$f = \frac{v_s}{2\pi} \sqrt{\frac{A}{V_o L}}$$

where:

- f = resonant frequency
- v_s = speed of sound in air ($343\,\frac{\text{m}}{\text{s}}$ at 20 °C and 1 atm)
- A = cross-sectional area of the neck of the bottle (m^2)
- V_o = volume of the main cavity of the bottle (m^3)
- L = length of the neck of the bottle (m)

(Note that it may be more convenient to use measurements in cm, cm^2, and cm^3, and use $v_s = 34\,300\,\frac{\text{cm}}{\text{s}}$.)

Blowing across the top of an open bottle is an example of a Helmholtz resonator.

You can make your mouth into a Helmholtz resonator by tapping on your cheek with your mouth open. You can change the pitch by opening or closing your mouth a little, which changes the area of the opening (A).

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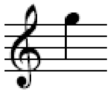
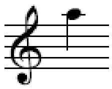
Frequencies of Music Notes

The frequencies that correspond with the pitches of the Western equal temperament scale are:

pitch	frequency (Hz)	pitch	frequency (Hz)
 A	440.0	 E	659.3
 B	493.9	 F	698.5
 C	523.3	 G	784.0
 D	587.3	 A	880.0

A note that is an octave above another note has exactly twice the frequency of the lower note. For example, the A in on the second line of the treble clef staff has a frequency of 440 Hz.* The A an octave above it (one ledger line above the staff) has a frequency of 440 × 2 = 880 Hz.

Harmonic Series

harmonic series: the additional, shorter standing waves that are generated by a vibrating string or column of air that correspond with integer numbers of half-waves.

fundamental frequency: the natural resonant frequency of a particular pitch.

harmonic: a resonant frequency produced by vibrations that contain an integer number of half-waves that add up to the half-wavelength of the fundamental.

The harmonics are numbered based on their pitch relative to the fundamental frequency. The harmonic that is closest in pitch is the 1st harmonic, the next closest is the 2nd harmonic, *etc.*

Any sound wave that is produced in a resonance chamber (such as a musical instrument) will produce the fundamental frequency plus all of the other waves of the harmonic series. The fundamental is the loudest, and each harmonic gets more quiet as you go up the harmonic series.

* Most bands and orchestras define the note "A" to be exactly 440 Hz, and use it for tuning.

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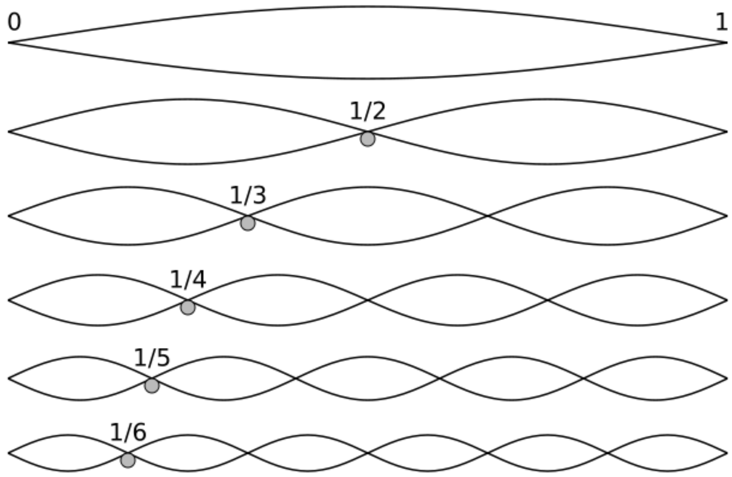
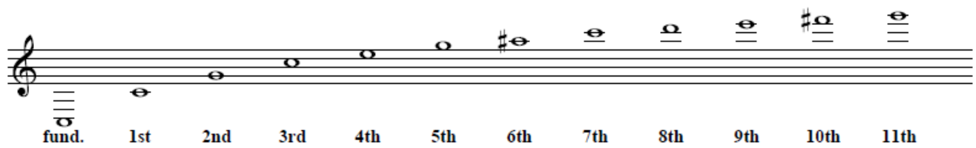
The following diagram shows the waves of the fundamental frequency and the first five harmonics in a pipe or a vibrating string:



Fraction of String	**Wave-length**	**Harmonic**	**Frequency**	**Pitch (relative to fundamental)**
1	$2L$	—	f_o	Fundamental.
$\frac{1}{2}$	$\frac{2L}{2}$	1st	$2f_o$	One octave above.
$\frac{1}{3}$	$\frac{2L}{3}$	2nd	$3f_o$	One octave + a fifth above.
$\frac{1}{4}$	$\frac{2L}{4}$	3rd	$4f_o$	Two octaves above.
$\frac{1}{5}$	$\frac{2L}{5}$	4th	$5f_o$	Two octaves + approximately a major third above.
$\frac{1}{6}$	$\frac{2L}{6}$	5th	$6f_o$	Two octaves + a fifth above.
$\frac{1}{n}$	$\frac{2L}{n}$	(n-1)th	nf_o	*etc.*



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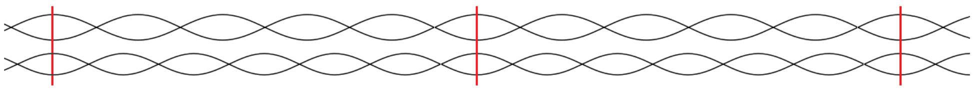
Beats

When two or more waves are close but not identical in frequency, their amplitudes reinforce each other at regular intervals.

For example, when the following pair of waves travels through the same medium, the amplitudes of the two waves have maximum constructive interference every five half-waves (2½ full waves) of the top wave and every six half-waves (3 full waves) of the bottom wave.



If this happens with sound waves, you will hear a pulse or "beat" every time the two maxima coïncide.

The closer the two wavelengths (and therefore also the two frequencies) are to each other, the more half-waves it takes before the amplitudes coïncide. This means that as the frequencies get closer, the time between beats gets longer.

Piano tuners listen for these beats, and adjust the tension of the string they are tuning until the time between beats gets longer and longer and finally disappears.

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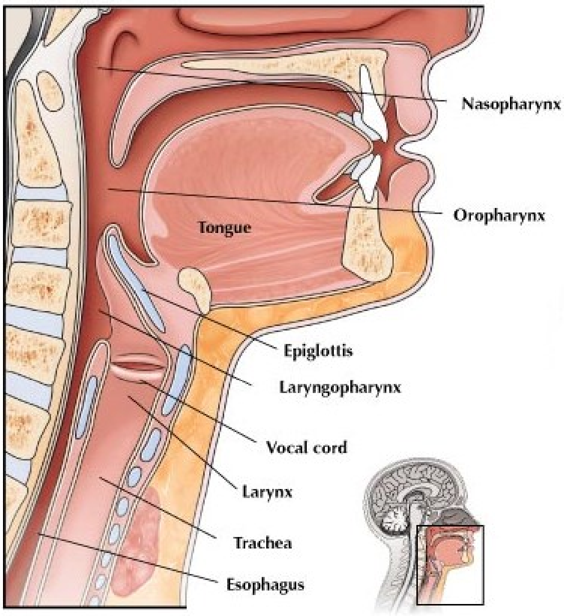
The Biophysics of Sound

When a person speaks, abdominal muscles force air from the lungs through the larynx.



The vocal cord vibrates, and this vibration creates sound waves. Muscles tighten or loosen the vocal cord, which changes the frequency at which it vibrates. Just like in a string instrument, the change in tension changes the pitch. Tightening the vocal cord increases the tension and produces a higher pitch, and relaxing the vocal cord decreases the tension and produces a lower pitch.

This process happens when you sing. Amateur musicians who sing a lot of high notes can develop laryngitis from tightening their laryngeal muscles too much for too long. Professional musicians need to train themselves to keep their larynx muscles relaxed and use other techniques (such as air pressure, which comes from breath support via the abdominal muscles) to adjust their pitch.

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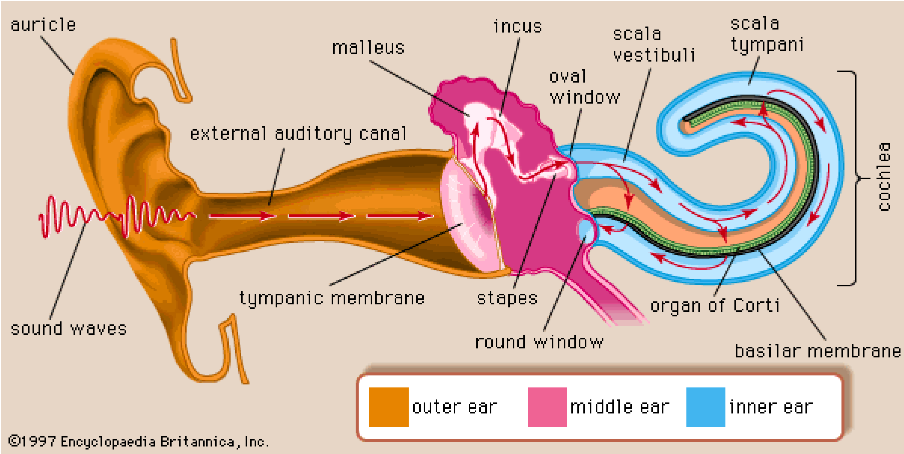
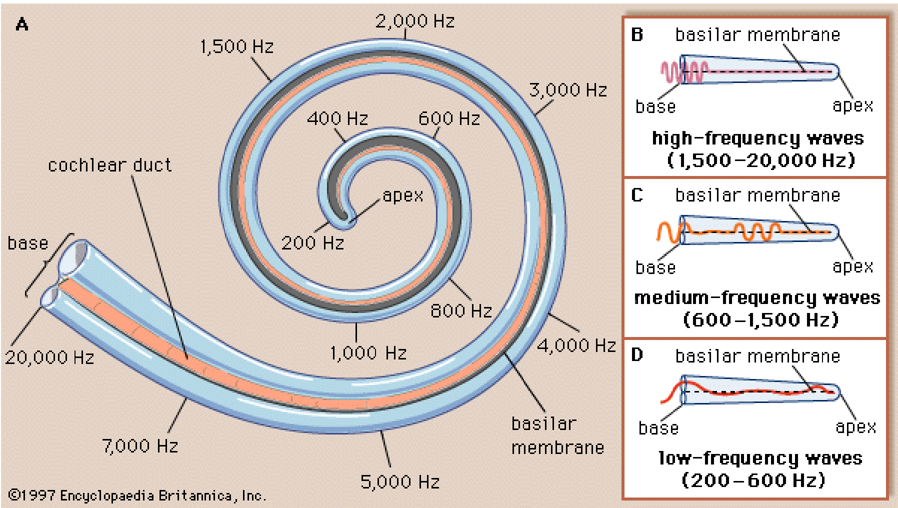
When the sound reaches the ears, it travels through the auditory canal and causes the tympanic membrane (eardrum) to vibrate. The vibrations of the tympanic membrane cause pressure waves to travel through the middle ear and through the oval window into the cochlea.



The basilar membrane in the cochlea is a membrane with cilia (small hairs) connected to it, which can detect very small movements of the membrane. As with a resonance tube, the wavelength determines exactly where the sound waves will vibrate the basilar membrane the most strongly, and the brain determines the pitch (frequency) of a sound based on the precise locations excited by these frequencies.



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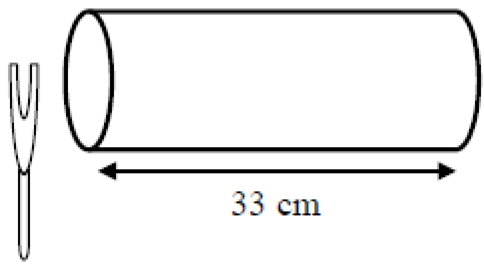
Homework Problems

A tuning fork is used to establish a standing wave in an open ended pipe filled with air at a temperature of 20°C, where the speed of sound is $343\,\frac{m}{s}$, as shown below:



The sound wave resonates at the 3nd harmonic frequency of the pipe. The length of the pipe is 33 cm.

1. **(M)** Sketch the pipe with the standing wave inside of it. (For simplicity, you may sketch a transverse wave to represent the standing wave.)

2. **(M)** Determine the wavelength of the resonating sound wave.

   Answer: 22 cm

3. **(M)** Determine the frequency of the tuning fork.

   Answer: 1 559 Hz

4. **(M)** What is the next higher frequency that will resonate in this pipe?

   Answer: 2 079 Hz